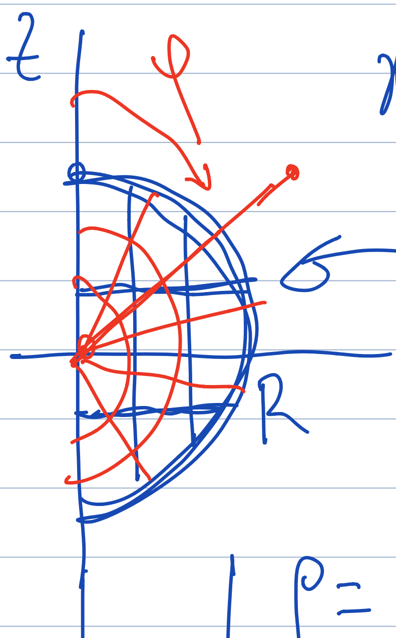
Aula Prática F7 + F8 26/4/21

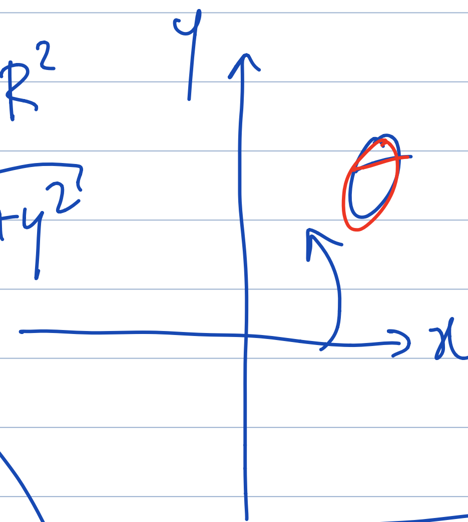
Ficha 7.

(ρ, θ, z) , (r, θ, φ)

$x^2+y^2+z^2 < R^2$



$\rho = \sqrt{x^2+y^2}$



$$\begin{cases} \rho = r \operatorname{sen} \varphi \\ z = r \cos \varphi \end{cases}$$

$r = \sqrt{x^2+y^2+z^2}$

$$\boxed{r^2 = \rho^2 + z^2}$$

$$\begin{cases} x = \rho \cos \theta \\ y = \rho \operatorname{sen} \theta \\ z = z \end{cases}$$

$\boxed{\rho}$

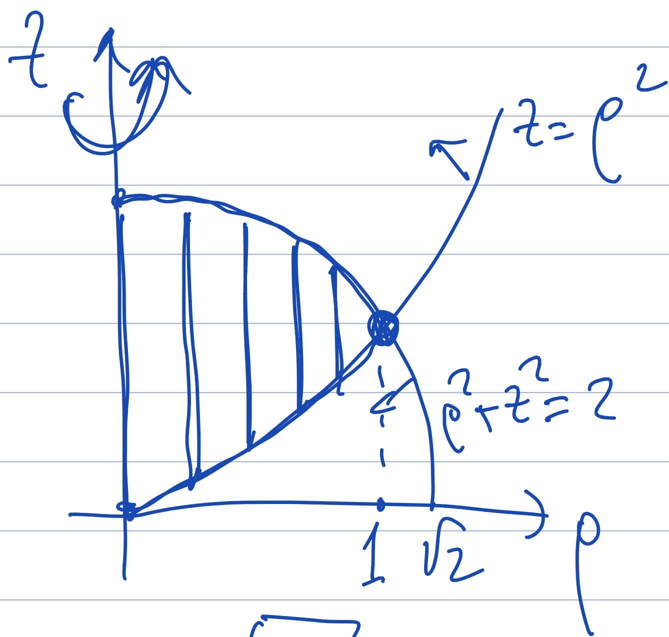
$$\begin{cases} x = r \operatorname{sen} \varphi \cos \theta \\ y = r \operatorname{sen} \varphi \operatorname{sen} \theta \\ z = r \cos \varphi \end{cases}$$

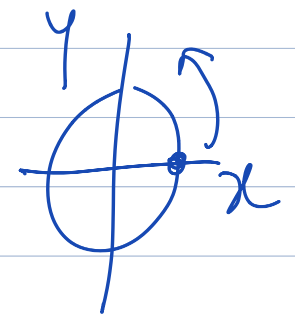
$\boxed{r^2 \operatorname{sen} \varphi}$

5-a) $\rho^2 < z < \sqrt{2-\rho^2}$

$z = \sqrt{2-\rho^2}$

$z^2 + \rho^2 = 2$



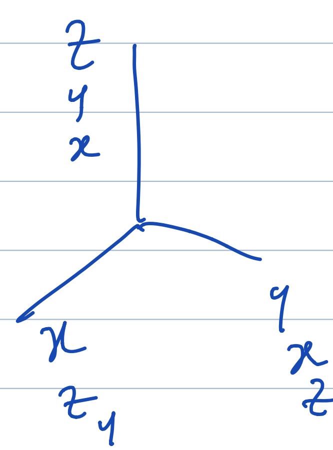


$0 < \theta < 2\pi$

$$\int_0^{2\pi} \left(\int_0^1 \left(\int_{\rho^2}^{\sqrt{2-\rho^2}} \rho \, dz \right) d\rho \right) d\theta$$

etc.

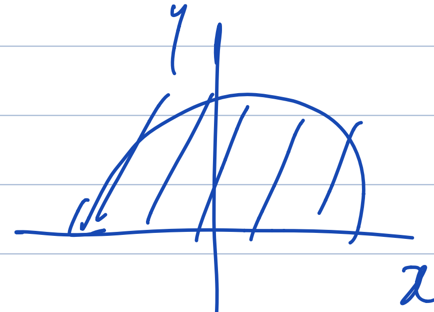
$$\boxed{\theta = \arctan \frac{y}{x}}$$



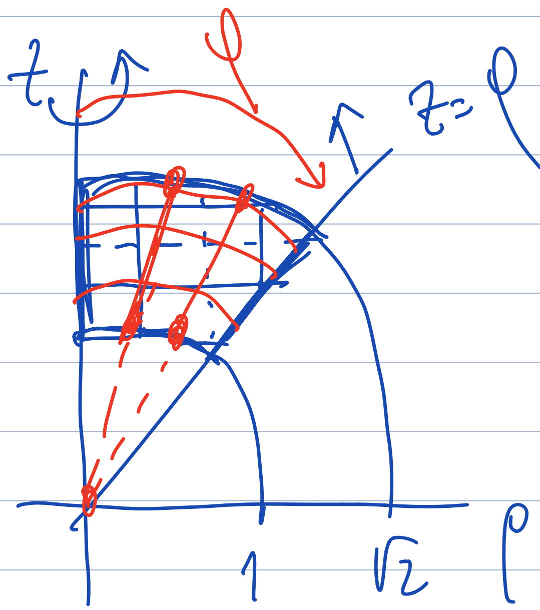
5-b)

$1 < \rho^2 + z^2 \le 2$;

$z > \rho$



$0 < \theta < \pi$



$1 < r < \sqrt{2}$

$0 < \varphi < \frac{\pi}{4}$

$r^2 \operatorname{sen} \varphi$

$$\int_0^{\pi} \left(\int_0^{\frac{\pi}{4}} \left(\int_1^{\sqrt{2}} r^2 \operatorname{sen} \varphi \, dr \right) d\varphi \right) d\theta$$

etc..

$X \subset \mathbb{R}^n$, $\quad f : \mathbb{R}^n \longrightarrow \mathbb{R}$

$$\int_X f$$

1 - $\boxed{f \equiv 1}$ $\quad \int_X 1 = vol_n(X)$.

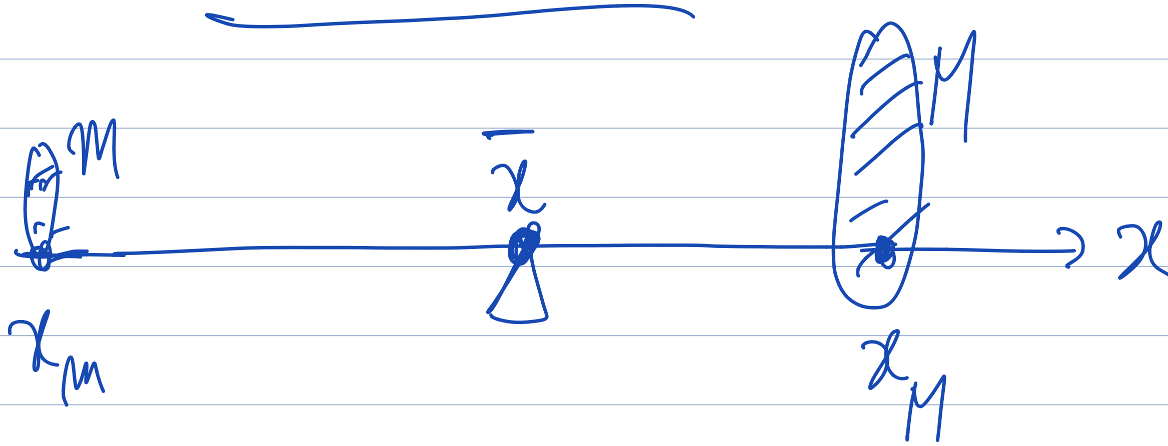
2 - $f \equiv$ densidade de massa

$f \geq 0$ $\qquad f = \sigma$ "sigma"

$$\frac{M}{V} = \sigma \iff M = \sigma V$$

$$= \int_X \sigma$$

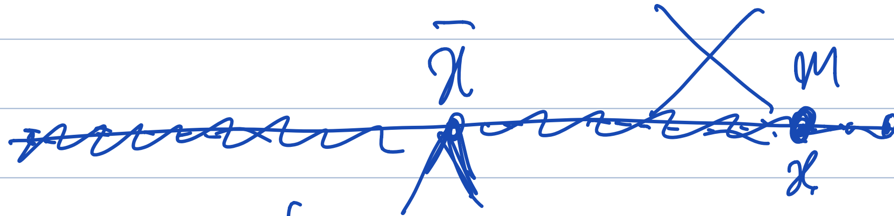
3- Centro de massa ou Centróide.



$$m(\bar{x} - x_m) = M(x_M - \bar{x})$$

$$(m + M)\bar{x} = M x_M + m x_m$$

$$\bar{x} = \frac{x_M M + x_m m}{\boxed{m + M}}$$

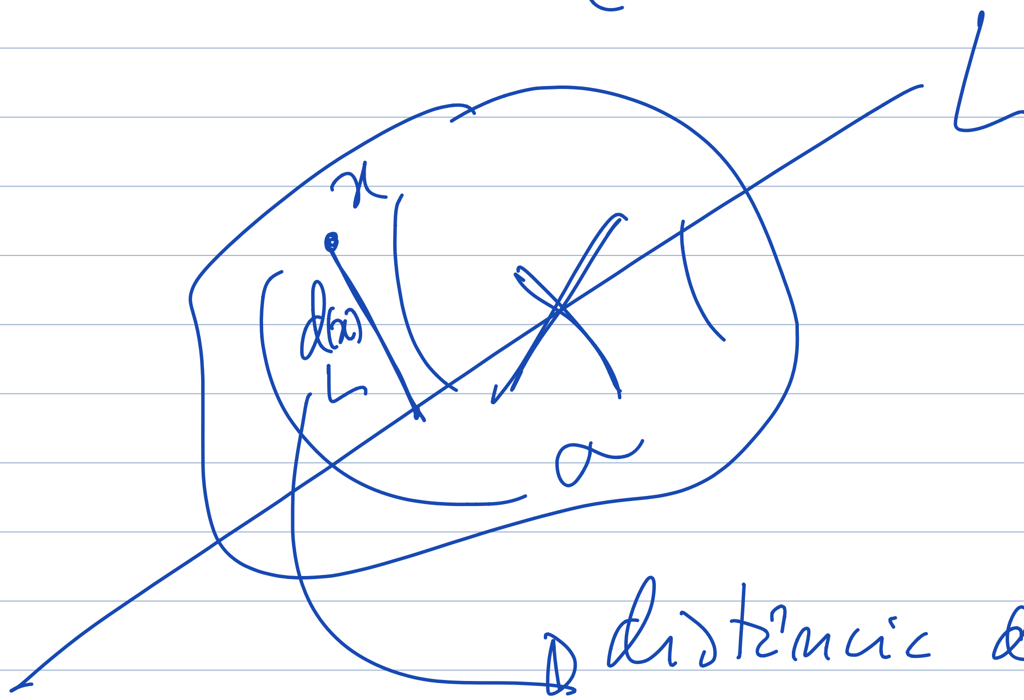


$$\bar{x} = \frac{\int_X x\,\sigma}{\int_X \sigma}$$

$$\boxed{\begin{gathered}\bar{x}_k = \frac{\int_X x_k\,\sigma}{\int_X \sigma} \\ k = 1, 2, \ldots, n\end{gathered}}$$

$n = 2, 3.$

4 – Momento de inércia relativo a um eixo (recta) L.



distância de x à linha recta L.

$$I_L(X) = \int_X \sigma \, d_L^2$$

$L =$ eixo Oz

$$d_L^2(x,y,z) = d_z^2(x,y,z) = x^2+y^2.$$

etc.

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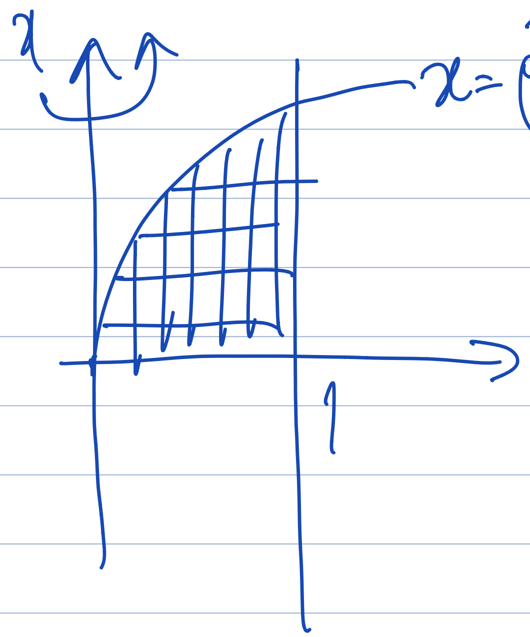
6 — X : $\sqrt{y^2+z^2} \leq 1$

$0 \leq x \leq (y^2+z^2)^{1/4}$

$\sigma(x,y,z) = x(y^2+z^2)$

$y \geq 0, \quad z \geq 0.$

$$I_x(X) = \int_X \sigma d_x^2 = ?$$



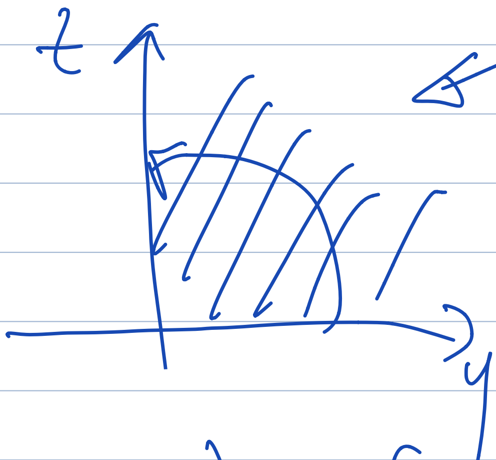
$x = \rho^{1/2} = \sqrt{\rho}$

(ρ, θ, x)

$\rho < 1$

$0 < x < \rho^{1/2}$

$\boxed{y > 0,\ z > 0}$



$0 < \theta < \frac{\pi}{2}$ ✓

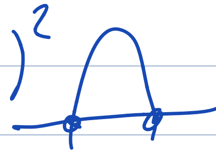
$$I_x(X) = \int_X \sigma\, d_x^2 = \int_X x \underbrace{(y^2+z^2)}_{\rho^2} \underbrace{(y^2+z^2)}_{\rho^2}$$

$$= \int_0^{\pi/2} \left(\int_0^1 \left(\int_0^{\sqrt{\rho}} \rho\, x \rho^2 \rho^2 \, dx \right) d\rho \right) d\theta$$

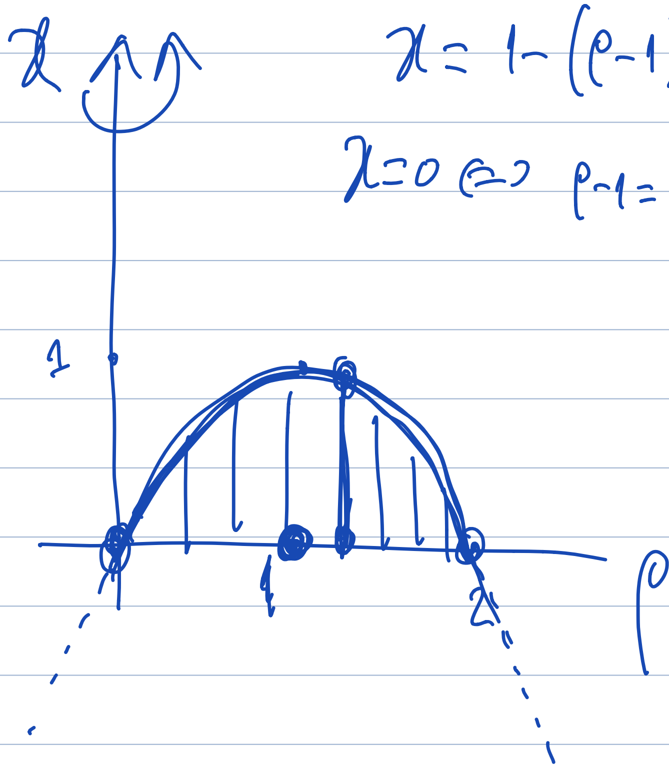
etc...

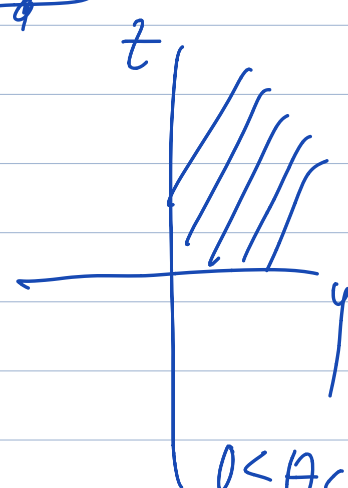
7-a) $\quad 0 \le x \le 1-(\rho-1)^2, \quad y \geq 0, \quad z \geq 0$

$x = 1-(\rho-1)^2$



$x=0 \iff \rho-1 = \pm 1$





$0<\theta<\frac{\pi}{2}$

$$\mathrm{Vol}_3(A) = \int_0^{\pi/2} \left(\int_0^{2} \left(\int_0^{1-(\rho-1)^2} \rho \, dz \right) d\rho \right) d\theta$$

etc. . . .

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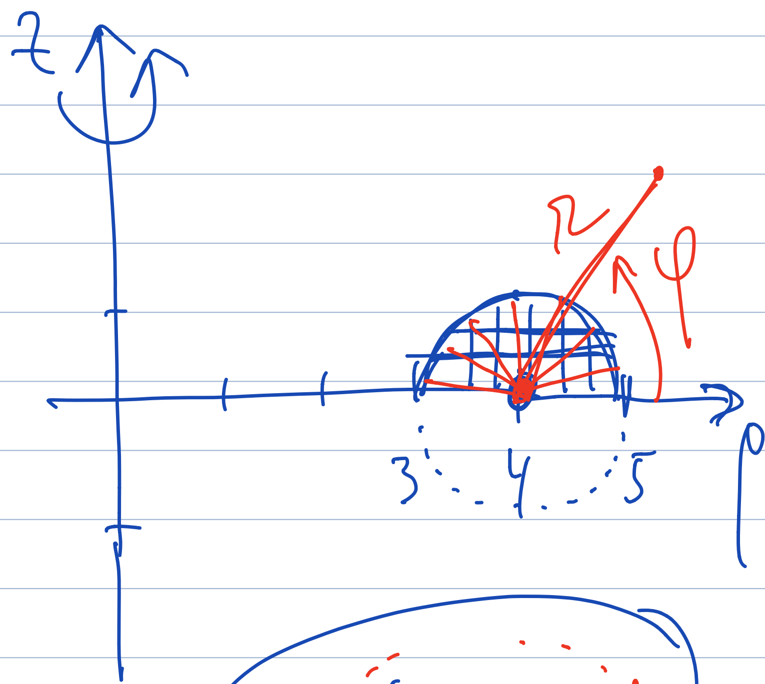
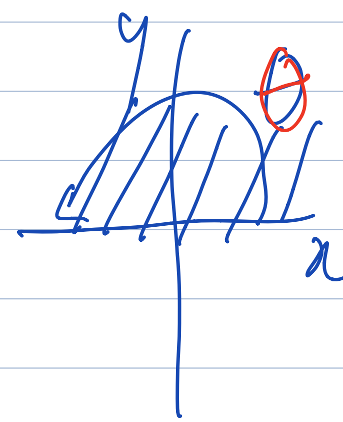
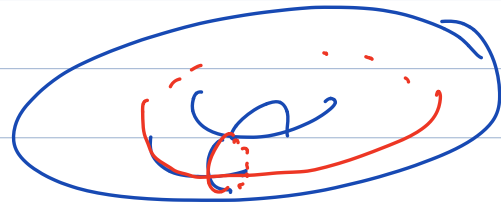
7-b) $(\rho-4)^2 + z^2 < 1, \quad y > 0$
$z > 0$





$0 < \theta < \pi$



Toro

Relação integral / derivada.

CDI-I: $\boxed{\text{TFC}}$

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f(t)\,dt = f(b(x))\,b'(x) - f(a(x))\,a'(x)$$

$$\frac{d}{dx}\int_{a}^{x} f(t)\,dt = f(x)$$

CDI-II: Regra de Leibniz:

$$\frac{d}{dt}\int_X f(x,t)\,dx = \int_X \frac{\partial f}{\partial t}(x,t)\,dx$$

8 - $F(t) = \int_0^1 \underbrace{\operatorname{sen}(tx^2 + x^3)}_{f(t,x)} \, dx$

Leibniz

$$F'(t) = \int_0^1 \frac{\partial f}{\partial t}(t,x)\,dx$$

$$F'(0) = \int_0^1 \frac{\partial f}{\partial t}(0,x)\,dx$$

$$\frac{\partial f}{\partial t}(t,x) = \cos(tx^2 + x^3)\, x^2$$

$$\frac{\partial f}{\partial t}(0,x) = x^2 \cos(x^3)$$

$$F'(0) = \frac{1}{3}\int_0^1 3x^2 \cos(x^3)\,dx = \frac{1}{3} \operatorname{sen}(x^3)\Big|_0^1$$

$$F'(0) = \frac{\operatorname{sen}(1)}{3}.$$

$$F(0) = \int_0^1 \operatorname{sen}(x^3)\,dx \quad \ldots$$

difícil.

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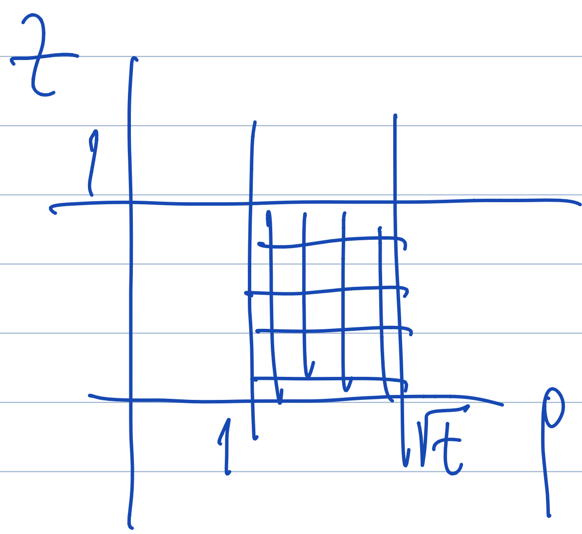
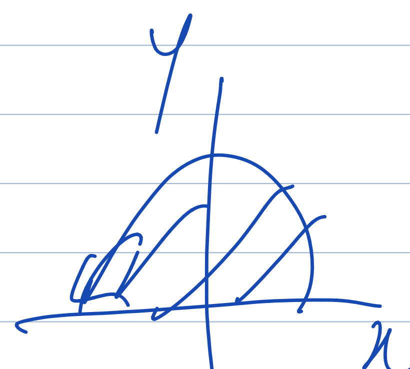
9 — $V_t: \begin{cases} 1 < x^2+y^2 < t \\ 0 < z < 1 \\ y > 0 \end{cases}$ (ρ, θ, z)





$0 < \theta < \pi$

$$f(t) = \int_{V_t} \frac{e^{t(x^2+y^2)}}{x^2+y^2}\, dx\, dy\, dz$$

não depende de θ nem de z.

$$= \int_1^{\sqrt{t}} \left(\int_0^1 \left(\int_0^{\pi} \rho \frac{e^{t\rho^2}}{\rho^2}\, d\theta \right) dz \right) d\rho$$

$$= \pi \int_1^{\sqrt{t}} \frac{e^{t\rho^2}}{\rho}\, d\rho = \pi \int_1^{b(t)} f(t,\rho)\, d\rho$$

TFC + R. Leibniz.

$$f(t,\rho) = \pi \frac{e^{t\rho^2}}{\rho}$$

$$F'(t) = \frac{d}{dt}\int_0^{b(t)} f(t,\rho)\,d\rho =$$

$$= f(t, b(t))\, b'(t) + \int_0^{b(t)} \frac{\partial f}{\partial t}(t,\rho)\,d\rho$$

$\underline{\text{Note}}$: $F(t) = I(b(t), t)$

$$F'(t) = \underbrace{\frac{\partial I}{\partial b} b'(t)}_{\text{TFC.}} + \underbrace{\frac{\partial I}{\partial t}}_{\text{Leibniz}}$$

$(t^{1/2})' = \frac{1}{2} t^{-\frac{1}{2}} = \frac{1}{2\sqrt{t}}$

$$F'(t) = \pi \frac{e^{t^2}}{2\sqrt{t}\sqrt{t}} + \frac{\pi}{2t}\int_0^{\sqrt{t}} \underbrace{2t e^{t\rho^2} \rho}\, d\rho$$

etc.